

Instructions: You can work on the problems in any order. Please use just one side of each page and clearly number the problems. You do not need to write answers on the question sheet.

This exam is a tool to help me (and you) assess how well you are learning the course material. As such, you should report enough written detail for me to understand how you are thinking about each problem.

You can use integration aids such as a table of integrals.

1. Determine the largest interval for which the existence-uniqueness theorem guarantees a unique solution to the following initial-value problem: (10 points)

$$y'' + \frac{1}{t^2 - 9}y' + \frac{1}{t}y = e^t, \quad y(1) = 3, \quad y'(1) = -7.$$

2. For each of the following, solve the given differential equation or initial-value problem. Express each result in terms of real-valued functions. (15 points each)

(a) $y'' + 4y' + 13y = 0, \quad y(0) = 4, \quad y'(0) = 2$

(b) $y'' - 5y = 7e^t + 4 \cos t$

(c) $\sin t \frac{d^2y}{dt^2} - \cos t \frac{dy}{dt} = \sin t$

3. Consider the differential equation $k^2y''(t) - 4ky'(t) - 7y(t) = 0$ where k is a positive constant.

(a) Find the general solution for this equation. (10 points)

(b) Describe the nature of the solution for large values of t . Oscillatory? Increasing? Decreasing? (5 points)

4. Show that

$$y(t) = c_1 \frac{1}{t^2} + c_2 t^3 + t^3 \ln t$$

is the general solution of the differential equation

$$t^2y'' - 6y = 5t^3$$

for the interval $(0, \infty)$. *You do not need to show where this formula comes from, just that it gives the general solution.* (15 points)

5. An object of mass m hangs on a spring with stiffness constant k . A dashpot connected to the mass provides a damping force proportional to the velocity of the object with proportionality constant γ . The dashpot is adjusted so that $\gamma = \sqrt{mk}$. The object is then displaced a distance y_0 from the equilibrium position and released from rest. Set-up and solve the relevant initial-value problem. Is this system underdamped, critically damped, or overdamped? (15 points)